

International GCSE Further Pure Mathematics – Paper 2 mark scheme

Paper 2					
Question	Working	Answer	Mark	AO	Notes
1	First term = $3e^{-1}$, Common Ratio = e^{-2} $S = \frac{3e^{-1}}{1-e^{-2}} \times \left(\frac{e^2}{e^2} \right) = \frac{3e}{e^2-1}$ (So, $a = 3$ and $b = 2$)	$S_{\infty} = \frac{3e}{e^2-1}$	B1B1 M1M1 A1 (5)	1 2	
2					
(a)	$3+x < 2x-1 \Rightarrow x > 4$	$x > 4$	B1	1	
(b)	$x(x-1) > 6 \Rightarrow x^2 - x - 6 > 0 \Rightarrow (x-3)(x+2) > 0$ critical values are $x = 3, -2$ so $x > 3$ OR $x < -2$	$x > 3$ OR $x < -2$	M1 M1A1	1	(Outside region)
(c)	$x > 3$ OR $x < -2$, $x > 4$ so $x > 4$	$x > 4$	B1 (5)	1	

Question	Working	Answer	Mark	AO	Notes
3 (a)	$\overline{AB} = -(4i+3j) + (8i+6j) = 4i + (p-3)j$ $ \overline{AB} = \sqrt{52} = \sqrt{4^2 + (p-3)^2} \Rightarrow p^2 - 6p - 27 = (p-9)(p+3) = 0$ $\Rightarrow p = 9, p = -3$ <p>(b)</p> $\overline{AB} = 4i + (p-3)j = 4i + 6j \Rightarrow \frac{1}{2\sqrt{13}}(4i+6j) = \frac{1}{\sqrt{13}}(2i+3j) \text{ (oe)}$	$\frac{1}{2\sqrt{13}}(4i+6j)$	B1 M1A1	1	
4 (a)	$f(-3) = 2 \times (-3)^2 + p \times (-3) + q \times (-3) + 12 = 0$ $\Rightarrow 42 = 9p - 3q \Rightarrow 14 = 3p - q$ $f'(x) = 6x^2 + 2px + q$ $f'(-3) = 6 \times (-3)^2 + 2p \times (-3) + q = 37$ $\Rightarrow -6p + q = -17 \Rightarrow q = 17 - 6p$ $14 = 3p - 1(6p - 17) \Rightarrow p = 1, q = -11$	$q = -11$	M1 M1 M1A1	3	Mark parts (i) and (ii) together
(b)	$\frac{(2x^3 + x^2 - 11x + 12)}{(x+3)} = 2x^2 - 5x + 4$ $\Rightarrow (x+3)(2x^2 - 5x + 4)$		M1A1	2	Solving simultaneous equations (by any method)
(c)	$b^2 - 4ac = (-5)^2 - 4 \times 2 \times 4 = -7 \text{ (7 hence no real roots for quadratic factor, so } x = -3 \text{ only real root.)}$		M1A1	1	
			(10)		

Question	Working	Answer	Mark	AO	Notes
5 (a)	$\cos(A - B) - \cos(A + B) = \cos A \cos B + \sin A \sin B - \cos A \cos B + \sin A \sin B$ $\Rightarrow 2 \sin A \cos B$		M1A1	1	
(b)	$2 \sin 5x \sin 3x = \cos 2x - \cos 8x$	$\cos 2x - \cos 8x$	B1	2	
(c)(i)	$\int 4 \sin 5\theta \sin 3\theta d\theta = 2 \int \cos 2\theta - \cos 8\theta d\theta = 2 \left[\frac{\sin 2\theta}{2} - \frac{\sin 8\theta}{8} \right] (+c)$		M1A1	3	
(ii)	$\int_0^{\frac{\pi}{6}} 4 \sin 5\theta \sin 3\theta dx = 2 \int_0^{\frac{\pi}{6}} (\cos 2\theta - \cos 8\theta) d\theta = 2 \left[\frac{\sin 2\theta}{2} - \frac{\sin 8\theta}{8} \right]_0^{\frac{\pi}{6}} = \frac{5\sqrt{3}}{8}$	$\frac{5\sqrt{3}}{8}$	M1A1 (7)		
6	$\log_x 2 = \frac{\log_2 2}{\log_2 x} = \frac{1}{\log_2 x}$ $\log_2 x + \frac{6}{\log_2 x} = 7 \Rightarrow (\log_2 x)^2 - 7 \log_2 x + 6 = 0$ $\Rightarrow (\log_2 x - 6)(\log_2 x - 1) = 0 \Rightarrow \log_2 x = 6, \log_2 x = 1$ $\Rightarrow x = 64, x = 2$	$64, 2$	B1 M1M1 A1 M1M1 A1 (7)	2, 3	

Question	Working	Answer	Mark	AO	Notes
7			M1A1	2	
(a)	$(i) 0 = \frac{ax-5}{x-b} \Rightarrow ax-5=0 \Rightarrow a\frac{5}{x}-5=0 \Rightarrow a=2$	$b=1$	B1	2	Accept $y=5$
(b)	$\text{When } x=0, y = \frac{-5}{-b} \Rightarrow y = \frac{5}{1} \Rightarrow y=5$	$(0, 5)$	B1	3	
(c)	$y=2$	$y=2$	B1	3	
(d)	<p>The graph shows a coordinate system with a vertical dashed line at $x=1$ and a horizontal dashed line at $y=2$. A curve approaches the vertical asymptote from the left, passing through the point $(0, 5)$. It then crosses the horizontal asymptote at $(2.5, 2)$ and continues to approach the vertical asymptote from the right.</p>		B1		Curve in correct quadrants
			B1 B1 (8)		Correct asymptotes drawn Correct intersections with axes

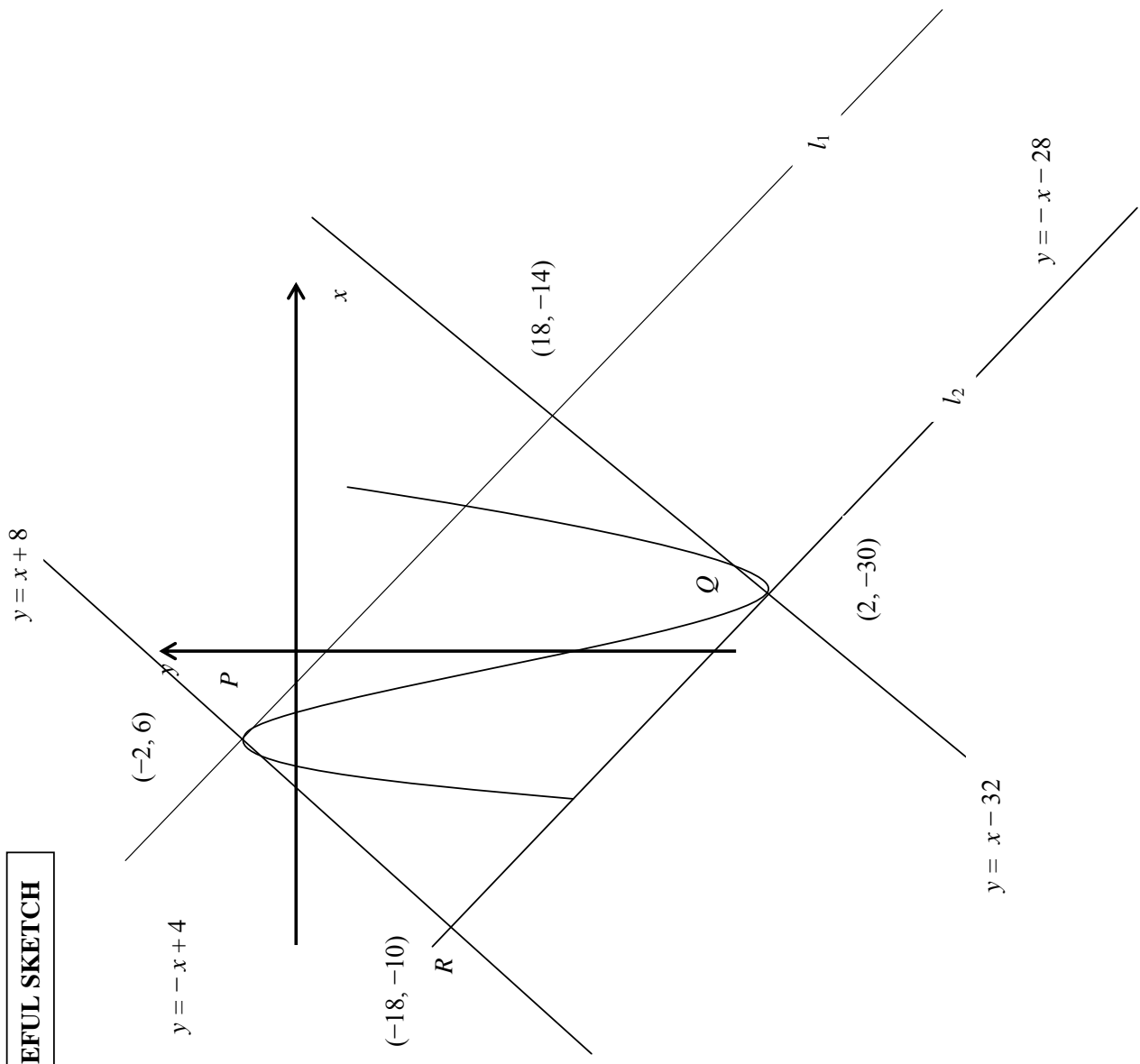
Question	Working	Answer	Mark	AO	Notes
8 (a)	$\frac{3}{\sqrt{1-2x}} = 3(1-2x)^{-\frac{1}{2}} \Rightarrow$ $= 3 \left\{ 1 + \left(-\frac{1}{2}\right)(-2x) + \left(-\frac{1}{2}\right)\left(\frac{3}{-2}\right)\frac{(-2x)^2}{2!} + \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{5}{-2}\right)\frac{(-2x)^3}{3!} + \dots \right\}$ $\frac{3}{\sqrt{1-2x}} = 3 + 3x + \frac{9}{2}x^2 + \frac{15}{2}x^3$		B1	2	
(b)	$-\frac{1}{2} < x < \frac{1}{2} \text{ or } x < \frac{1}{2}$	$-\frac{1}{2} < x < \frac{1}{2}$	B1	2	
(c)	$\frac{3}{\sqrt{0.9}} = \frac{3}{\sqrt{\frac{9}{10}}} = \frac{3}{\frac{3}{\sqrt{10}}} = \sqrt{10}$		B1	3	
(d)	$\frac{1}{\sqrt{10}-3} \times \frac{(\sqrt{10}+3)}{(\sqrt{10}+3)} = \frac{(\sqrt{10}+3)}{1} = (\sqrt{10}+3)$	$(\sqrt{10}+3)$	M1A1	3	
(e)	$1-2x=0.9 \Rightarrow x=0.05$ $\sqrt{10}+3 = 3+3 \times (0.05) + \frac{9}{2} \times (0.05)^2 + \frac{15}{2} \times (0.05)^3 + \dots + 3 = 3.1621875\dots + 3$ ≈ 6.16219	6.16219	B1 M1A1 (11)	3	

Question	Working	Answer	Mark	AO	Notes
9 (a)	$7 + 4x - 2x^2 = -2[(x-1)^2 - 1] + 7 = -2(x-1)^2 + 9$ $P = -2, Q = -1, R = 9$		M1A1 A1	1	
(b)	(i) 9 (ii) 1	(i) 9 (ii) 1	B1B1	2	
(c)	$7 + 4x - 2x^2 = 4 - x \Rightarrow 2x^2 - 5x - 3 = 0$ $\Rightarrow (2x+1)(x-3) = 0 \Rightarrow x = -\frac{1}{2}, 3$	$x = -\frac{1}{2}, x = 3$	M1M1 A1	2	
(d)	$V = \pi \int_{-0.5}^3 (7 + 4x - 2x^2)^2 dx = \pi \int_{-0.5}^3 (4-x)^2 dx$ $\Rightarrow \pi \int_{-0.5}^3 33 + 64x - 13x^2 - 16x^3 + 4x^4 dx$ $\Rightarrow \pi \left[33x + 32x^2 - \frac{13x^3}{3} - 4x^4 + \frac{4x^5}{5} \right]_{-0.5}^3 = \pi \frac{4459}{30} = 466.945 \dots \approx 467$	467	M1 M1A1 M1A1	3	
			(13)		

Question	Working	Answer	Mark	AO	Notes
10 (a)	$y = f(-2) = 6$ $\frac{dy}{dx} = 3x^2 - 13 \Rightarrow f'(-2) = 13 - 13 = -1$ Equation of tangent $y - 6 = -1(x - (-2)) \Rightarrow y + x - 4 = 0$	B1	B1	1	
(b)	$-1 = 3x^2 - 13 \Rightarrow 3x^2 - 12 = 0 \Rightarrow x^2 - 4 = 0 \Rightarrow (x - 2)(x + 2) = 0$ at $Q, x = 2$ $f(2) = -30$ $y - 30 = -(x - 2) \Rightarrow \{y = -x - 28\}$	M1A1	M1A1	1, 2	
(c)	Gradient of normal = 1 Equation of Normal $y - 6 = x - (-2) \Rightarrow \{y = x + 8\}$ At $R,$ $x + 8 = -x - 28 \Rightarrow x = -18$ When $x = -18, y = -10$	B1	B1	2	
		$(-18, -10)$	A1A1		

Question	Working	Answer	Mark	AO	Notes
10 cont'd (d)	<p>{Coordinates of R $(-18, -10)$}</p> $PR = \sqrt{(6 - -10)^2 + (-2 - -18)^2} = 16\sqrt{2}$	$16\sqrt{2}$	M1A1	3	
(e)	<p>Area of rectangle</p> <p>Length of $QR = \sqrt{(-18 - 2)^2 + (-10 - -30)^2} = 20\sqrt{2}$</p> <p>Area = $16\sqrt{2} \times 20\sqrt{2} = 640$</p>	640	M1 M1A1	3	
(e)	<p>ALTERNATIVE</p> <p>Equation of normal at Q</p> $y - -30 = x - 2 \Rightarrow y = x - 32$ <p>Coordinates of 4th vertex of rectangle</p> $x - 32 = -x + 4 \Rightarrow x = 18 \text{ so } y = -14$ $\text{Area} = \frac{1}{2} \begin{vmatrix} -2 & -18 & 2 & 18 & -2 \\ 6 & -10 & -30 & -14 & 6 \end{vmatrix} = 640$	640	M1 M1A1 (18)	3	

USEFUL SKETCH



Question	Working	Answer	Mark	AO	Notes
11	<p>Total Surface Area of the cone</p> $l = \frac{x}{\sin 30^\circ} = 2x$ $A = \pi r l + \pi r^2 \Rightarrow \pi x \times 2x + \pi x^2 = 3\pi x^2$ <p>Volume of cone</p> $h = \frac{x}{\tan 30^\circ} = \sqrt{3}x$ $\text{Vol} = \frac{1}{3} \pi r^2 h \Rightarrow V = \frac{\sqrt{3}}{3} \pi x^3$ $\frac{dA}{dt} = 10 \text{ (cm}^2 \text{ / s)} \quad \frac{dV}{dx} = \sqrt{3} \pi x^2, \frac{dA}{dx} = 6\pi x$ $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} \times \frac{dA}{dA}$ $\Rightarrow \frac{dV}{dt} = \sqrt{3} \pi x^2 \times 10 \times \frac{1}{6\pi x} = \frac{5\sqrt{3}}{3} x \Rightarrow 10\sqrt{3} \text{ (cm}^3 \text{ / s)}$		<p>B1</p> <p>M1A1</p> <p>B1</p> <p>M1A1</p> <p>B1M1</p> <p>M1</p> <p>M1A1</p> <p>(11)</p> <p>100</p>	2, 3	M1 for differentiating either $\frac{dV}{dx}$ or $\frac{dA}{dx}$
		Total	100		